Affine adjustable robust optimization in unit commitment and economic dispatch problems

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1. Adjustable robust optimization

Framework of ARO

$$\min\{\boldsymbol{c}^T\boldsymbol{x} \mid \forall \boldsymbol{w}\in W, \exists \boldsymbol{y}: \boldsymbol{Ax}+\boldsymbol{By}\le \boldsymbol{b}^0-\boldsymbol{Cw}\}$$

- Dispatch process is divided into two stages
- $\boldsymbol{x}$ is here-and-now decision which is made before uncertainty $\boldsymbol{w}$ is known and must ensure that there will be at least one wait-and-see decision $\boldsymbol{y}$ such that operating constraints are satisfied after $\boldsymbol{w}$ realizes.
- Feasibility can be recovered for any $\boldsymbol{w}$ by adjusting $\boldsymbol{y}$. This is the reason it is called adjustable / adaptive / recoverable.
- ARO is NP-hard to solve
- If $\boldsymbol{y}$ is linearly dependent on $\boldsymbol{w}$, ARO is called AARO

$$\boldsymbol{y}(\boldsymbol{w})=\boldsymbol{y}^0+\boldsymbol{Gw}$$

2. A zero-sum game interpretation

- Under affine feedback policy, AARO can be viewed as a zero-sum game between the decision maker and the uncertainty

$$\max_{w} \min_{\boldsymbol{x},\boldsymbol{y}^0,\boldsymbol{G}} \{\boldsymbol{c}^T \boldsymbol{x} + \boldsymbol{d}^T \boldsymbol{y}^0 \mid \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{y}^0 + \boldsymbol{G}\boldsymbol{w}) \le \boldsymbol{b}^0 - \boldsymbol{C}\boldsymbol{w},\ \boldsymbol{w} \in W\}$$

- $\boldsymbol{y}$ is regarded as feedback with respect to $\boldsymbol{w}$
- Decision maker seek for best $\boldsymbol{x}$, $\boldsymbol{y}^0$ as well as feedback gain matrix $\boldsymbol{G}$ (very similar to robust control). Strategy $\boldsymbol{x}$ is reliable against the worst disturbance while minimizes the cost under nominal scenario (uncertainty is equal to forecast)
- The uncertainty $\boldsymbol{w}$ tries to attack the system in a worst manner, in terms of feasibility.

3. Equivalent LP of AARO

$$(\boldsymbol{BG}+\boldsymbol{C})\boldsymbol{w} \le \boldsymbol{b}^0 - \boldsymbol{Ax} - \boldsymbol{By}^0 \quad \forall \boldsymbol{w} \in W$$

$$\max_{\boldsymbol{w}\in W}(\boldsymbol{BG}+\boldsymbol{C})_i \boldsymbol{w} \le (\boldsymbol{b}^0 - \boldsymbol{Ax} - \boldsymbol{By}^0)_i, \quad \forall i$$

$$W = \{\boldsymbol{w} \mid \boldsymbol{Fw} \le \boldsymbol{f}\}$$

$$\begin{array}{l} \exists \Gamma_i \ge 0,\ \Gamma_i \boldsymbol{F} = (\boldsymbol{BG}+\boldsymbol{C})_i \\ \Gamma_i \boldsymbol{f} \le (\boldsymbol{b}^0 - \boldsymbol{Ax} - \boldsymbol{By}^0)_i \end{array} \quad \forall i$$

$$\Gamma \ge 0,\ \Gamma \boldsymbol{F} = \boldsymbol{BG}+\boldsymbol{C}$$

$$\Gamma \boldsymbol{f} \le \boldsymbol{b}^0 - \boldsymbol{Ax} - \boldsymbol{By}^0$$

3. Equivalent LP of AARO

$$\max_{w} \min_{\boldsymbol{x},\boldsymbol{y}^0,\boldsymbol{G}} \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}^0 \mid \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{y}^0 + \boldsymbol{G}\boldsymbol{w}) \le \boldsymbol{b}^0 - \boldsymbol{C}\boldsymbol{w},\ \boldsymbol{w} \in W\}$$

$$\Downarrow$$

$$\min_{\boldsymbol{x},\boldsymbol{y}^0,\boldsymbol{G}} \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}^0$$

$$s.t.\ \ \boldsymbol{x} \in X,\ \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y}^0 \le \boldsymbol{b}^0$$

$$\text{vec}(\Gamma) \ge 0,\ \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y}^0 + (\boldsymbol{f}^T \otimes \boldsymbol{I}_m)\text{vec}(\Gamma) \le \boldsymbol{b}^0$$

$$(\boldsymbol{F}^T \otimes \boldsymbol{I}_m)\text{vec}(\Gamma) = (\boldsymbol{I}_n \otimes \boldsymbol{B})\text{vec}(\boldsymbol{G}) + \text{vec}(\boldsymbol{C})$$

4. Applications

Joint energy and reserve scheduling (ED)

p_i^f current output of generator i

r_i reserve capacity of generator i

p_i^c output of generator i after corrective action

p_j^w output of wind farm j

Task: allocate reserve capacity in generating units such that in a dispatch interval, wind generation uncertainty can be mitigated by only deploying available reserve resources.

4. Applications

Joint energy and reserve scheduling (ED)

$$\max_{p_j^w \in P^W} \min_{p_i^f, r_i, g_{ij}} F = \sum_{i=1}^{N} \left(a_i \left(p_i^f \right)^2 + b_i p_i^f + c_i r_i \right)$$

$$s.t.\ p_i^f + r_i \le P_{\max}^i,\ P_{\min}^i \le p_i^f - r_i,\ 0 \le r_i \le \min\left\{ R_i^-, R_i^+ \right\}\ \forall i$$

$$\sum_i p_i^f + \sum_j p_j^{we} = \sum_q p_q$$

$$\left| \sum_i \pi_{il} p_i^f + \sum_j \pi_{jl} p_j^{we} - \sum_q \pi_{ql} p_q \right| \le F_l\ \forall l$$

$$-r_i \le p_i^c - p_i^f \le r_i,\ P_{\min}^i \le p_i^c \le P_{\max}^i \forall i$$

$$\sum_i p_i^c + \sum_j (p_j^{we} + p_j^w) = \sum_q p_q$$

$$\left| \sum_i \pi_{il} p_i^c + \sum_j \pi_{jl} (p_j^{we} + p_j^w) - \sum_q \pi_{ql} p_q \right| \le F_l\ \forall l$$

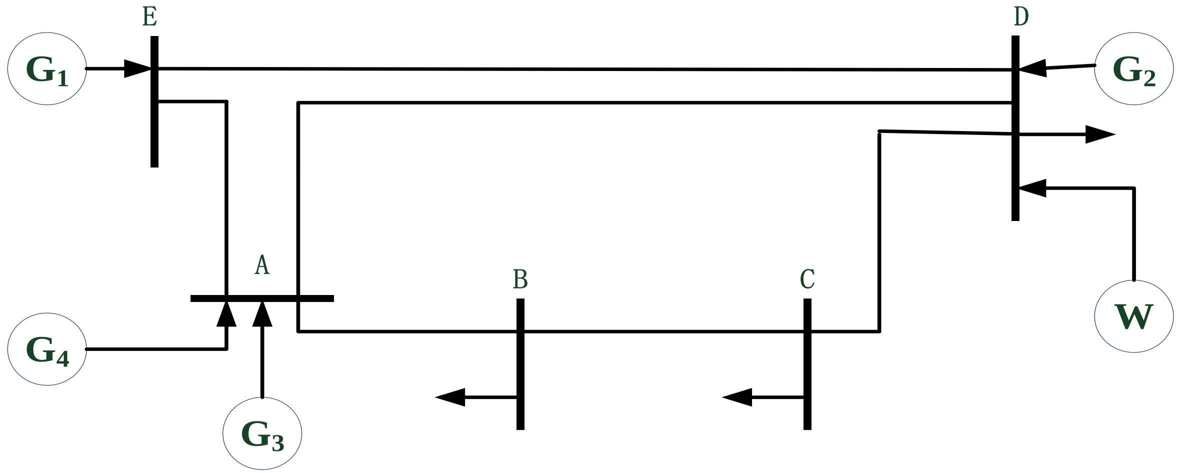
$$p_i^c = p_i^f + \sum_j g_{ij} p_j^w\ \forall i$$

$$P^W = \left\{ p^w \middle| \begin{array}{c} p_j^{wl} \le p_j^w \le p_j^{wu}\ \forall j \\ \sum_j \left| p_j^w - p_j^{we} \right| / p_j^{wh} \le \Gamma^S \end{array} \right\}, \Gamma^S = M\mu + \Phi^{-1}(\alpha)\sqrt{M}\sigma$$

4. Applications

Joint energy and reserve scheduling (ED)

Five bus system



Generator data

Unit NO.	P_{min}/P_{max} MW	Offering Price ￥/MWh	Reserve Price ￥/MWh	Ramp MW/h
G_1	[180, 400]	200	300	50
G_2	[100, 300]	300	450	50
G_3	[150, 600]	360	540	100
G_4	[120, 500]	250	400	80

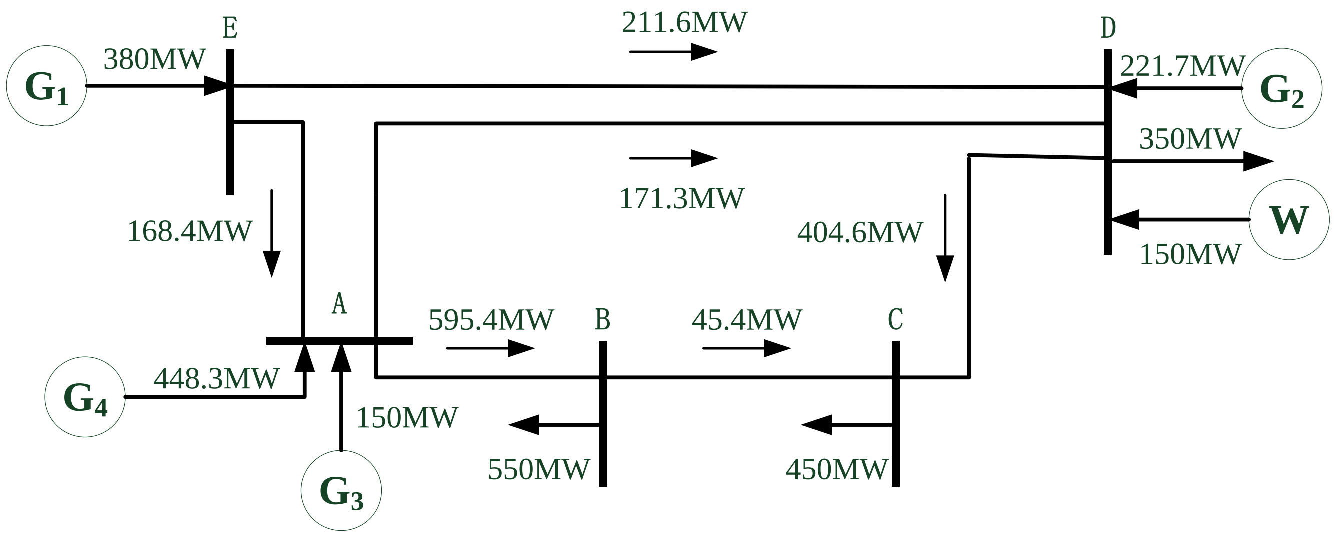
Transmission line data

Line NO.	From Node	To Node	Reactance p.u.	Transmission Limit MW
L_1	A	B	0.0281	600
L_2	A	D	0.0304	300
L_3	A	E	0.0064	200
L_4	B	C	0.0108	300
L_5	C	D	0.0297	420
L_6	D	E	0.0297	300

4. Applications

Joint energy and reserve scheduling (ED)

- 20MW wind generation uncertainty
- Reserve is assigned via AARO model



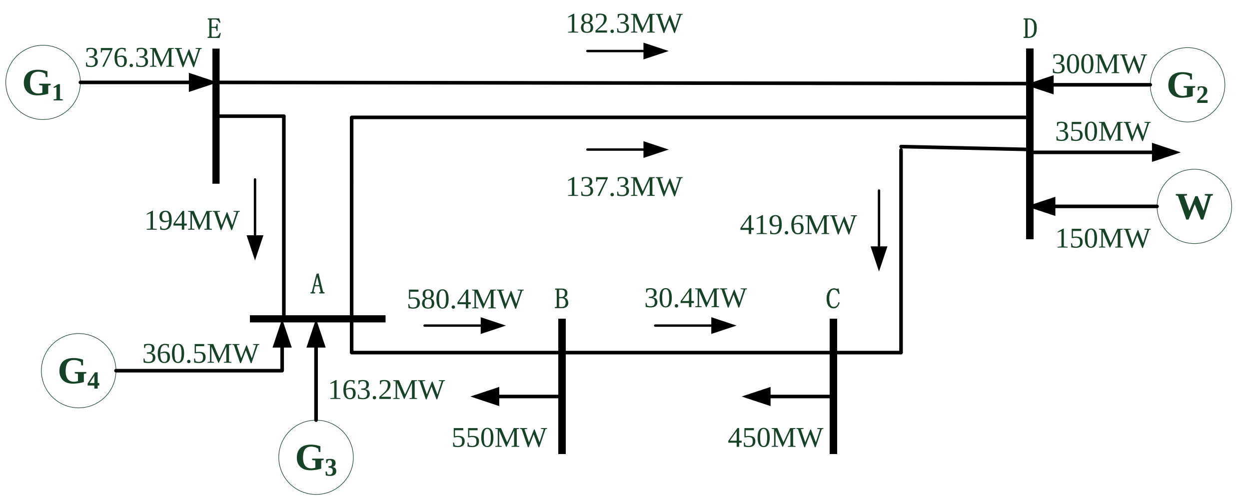
	G_1	G_2	G_3	G_4
Output / MW	380	221.7	150	448.3
Reserve / MW	20	0	0	0
Dispatch Cost / ￥	308,584			
Reserve Cost / ￥	6,000			
Total Cost/ ￥	314,584			

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 380 \\ 221.7 \\ 150 \\ 448.3 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} p^w$$

4. Applications

Joint energy and reserve scheduling (ED)

- 100MW wind generation uncertainty
- Reserve is assigned via AARO model



	G_1	G_2	G_3	G_4
Output / MW	376.3	300	163.2	360.5
Reserve / MW	6.8	13.2	0	80
Dispatch Cost /￥	314,138			
Reserve Cost /￥	41,173			
Total Cost /￥	355,311			

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 376.3 \\ 300.0 \\ 163.2 \\ 360.5 \end{bmatrix} + \begin{bmatrix} -0.068 \\ -0.132 \\ 0 \\ -0.800 \end{bmatrix} p^w$$

4. Applications

Unit commitment with wind generation

$$\min_{\beta,u,p^0,G} \max_{p^w} \sum_{t=1}^{T}\sum_{i=1}^{N}\left(S_i\beta_{it} + c_i u_{it} + b_i p_{it}^0\right)$$

$$s.t.\quad -u_{i(t-1)} + u_{it} - u_{ik} \le 0,\; u_{i(t-1)} - u_{it} + u_{ik} \le 1, -u_{i(t-1)} + u_{it} - \beta_{it} \le 0, \forall i,\ \forall t,\ \forall k$$

$$\sum_{i=1}^{N} u_{it}P_{\max}^{i} \ge (1+r)D_t,\; \sum_{i=1}^{N} p_{it} + \sum_{j=1}^{M} p_{jt}^{w} = D_t, \forall t, \forall\{p_{jt}^{w}\} \in P^{W}$$

$$P_{\min}^{i}u_{it} \le p_{it} \le P_{\max}^{i}u_{it}, p_{i(t+1)} - p_{it} \le R_{+}^{i},\; p_{it} - p_{i(t+1)} \le R_{-}^{i},\ \forall i, \forall t$$

$$\left|\sum_{i=1}^{N}\pi_{il}p_{it} + \sum_{j=1}^{M}\pi_{jl}p_{jt}^{w} + \sum_{q=1}^{Q}\pi_{jl}p_{qt}\right| \le F_l \quad \forall l, \forall t, \forall\{p_{jt}^{w}\} \in P^{W}$$

$$p_{it} = p_{it}^{0} + \Delta p_{it}, p_{jt}^{w} = p_{jt}^{e} + \Delta p_{jt}, \Delta p_{it} = \sum_{j=1}^{M} g_{ij}\Delta p_{jt} \quad \forall i, \forall t, \forall\{p_{jt}^{w}\} \in P^{W}$$

$$P^{w} = \{\{p_{jt}^{w}\} \mid p_{jt}^{l} \le p_{jt} \le p_{jt}^{u}, \sum\nolimits_{j=1}^{M}\left|p_{jt} - p_{jt}^{e}\right| / p_{jt}^{h} \le \Gamma^{S}, \forall t, \sum\nolimits_{t=1}^{T}\left|p_{jt} - p_{jt}^{e}\right| / p_{jt}^{h} \le \Gamma^{T}, \forall j\}$$

4. Applications

Unit commitment with wind generation

A real power system in China

- 177 thermal units, 1880 buses;
- 5 virtual wind farms in planning

- Computational performance

Γ^S	Γ^T	UC Cost / ￥	Total Cost / ￥	Computing Time/s
0	0	7.395×10^7	5.063×10^8	201
0.5	1.7	7.422×10^7	5.077×10^8	487
1.0	3.4	7.550×10^7	5.094×10^8	514
1.5	5.1	7.859×10^7	5.129×10^8	550
2.0	6.8	7.994×10^7	5.145×10^8	586
2.5	8.5	8.035×10^7	5.161×10^8	605

Thank you and questions